

**Abstract.** *This paper analyses the efficiency or productivity at the level of a production unit, but also at industry level, resorting for this purpose to both parametric and non-parametric techniques. Cost function model specifications are described herein, considering that the technical inefficiency effects determine the companies to operate below the production stochastic frontier.*

*Also, the estimations of the production frontier are rendered by means of Cobb-Douglas, C.E.S. and translog functions, evidencing that the latter is the most flexible of them. On the other hand, in many respects, C.E.S. production function is more appropriate to reality as compared to Cobb-Douglas function. At the same time, the estimation of C.E.S. function parameters is more difficult, Cobb-Douglas function being, in this respect, preferred.*

*The last part of the paper is consecrated to the study of efficiency at industry level and to conclusions. The Input variables used within the analysis are: fixed assets, inventories, number of employees, while the Output ones are: operating revenues and net profit.*

*As for the conclusions, the results reveal that the efficiency level for several companies is quite low. Therefore, a deeper interest should be manifested in order to increase the efficiency level in the construction industry.*

**Keywords:** cost function industry, Data Envelopment Analysis, production function, stochastic frontier.

## **STOCHASTIC FRONTIER ANALYSIS OF PRODUCTION FUNCTION AND COST FUNCTION ESTIMATION METHODS. STUDY OF EFFICIENCY AT INDUSTRY LEVEL**

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## 1. Introduction

In analysing the efficiency or productivity of a production unit, we may use the distance function for both inputs and outputs. This allows us to compute the “radial” distance of the production unit in relation to the production function, the important issue being to *estimate such production frontier*. For this end, we may start from the idea that the production theory has revealed classes of production functions, depending on many parameters, functions corresponding to the transformation of inputs into outputs. *Cobb-Douglas* production functions with three parameters, *translog* production function including also the time parameter etc. belong to such classes.

Therefore, in case of a homogenous production unit group, we should identify first of all the *class of production functions* corresponding to the internal process of transformation of inputs into outputs and then *to estimate the appropriate parameters*.

As for the efficiency measurement, this could be done by appealing to both parametric and non-parametric techniques.

For the first case, we could state that the units for which we do have observed values on inputs and outputs form a sample. By using econometric techniques, we will estimate all parameters of the selected model and, for each unit of the sample, we will also estimate its distance to the production frontier.

For the *non-parametric techniques* measuring the distance up to the production frontier, which approximate the frontier by creating an *envelope* of the input and output variables corresponding to a scale yield, linear and/or non-linear mathematic programming models are used.

The efficiency analysis is not a recent topic of interest for economists, its roots coming from Knight, in 1933. In 1951, Debreu and Koopmans have presented the results of their studies regarding the efficiency computing. Schmidt (1977), Olsen et al. (1980), Forsund et al. (1980), Forsund and Hjalmarsson (1987), Lovell și Schmidt (1988), Greene (1993), Cooper et al. (2007), Zhu (2009) and others have brought important contributions to the efficiency study, by using both parametric and non-parametric methods.

## 2. Model specifications

The stochastic frontier of the production function has been independently proposed by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977). The original specification implies a production function generated for a cross-sectional data set, with a two-component error term, one relating to stochastic effects and the other one to technical inefficiency.

This model may be rendered under the following form:

$$Q_i = \alpha u_i + (Z_i - X_i), \text{ cu } i = \overline{1, n} \quad (1)$$

where:

$Q_i$  – production (or production logarithm) of company  $i$ ;

$u_i$  – vector of type  $k \times 1$ ; it represents the input quantities of company  $i$ ;

$\alpha$  – vector of unknown parameters;

$Z_i$  – stochastic variables considered  $N(0, \sigma_Z^2)$  and independent from  $X_i$ ;

$X_i$  – non-negative stochastic variables relating to production technical inefficiency and considered  $|N(0, \sigma_X^2)|$ .

Various authors have consistently contributed to this area of interest, among them: Forsund, Lovell and Schmidt (1980), Schmidt (1986), Bauer (1990) and Greene (1993), Cooper et al. (2007), Zhu (2009).

FRONTIER 4.1 is a tool allowing maximum probability estimates of a subset of the stochastic frontier production and of the cost functions proposed in the related literature.

FRONTIER 4.1 has been conceived to estimate the specifications of the model detailed in Battese and Coelli (1988, 1992 and 1995) and Battese, Coelli and Colby (1989). Since then, the specifications in Battese and Coelli (1988) and Battese, Coelli and Colby (1989) are particular cases of the Battese și Coelli (1992) specification.

#### **Model 1: Battese and Coelli (1992) Specification**

Battese and Coelli (1992) propose a stochastic frontier production function.

The model can be rendered as follows:

$$Q_{it} = \alpha u_{it} + (Z_{it} - X_{it}), \text{ with } i = \overline{1, n} \text{ and } t = \overline{1, T} \quad (2)$$

where:

$Q_{it}$  – production logarithm at the level of company  $i$  at time  $t$ ;

$u_{it}$  – vector of type  $k \times 1$ ; it represents the input quantities (transformations) of company  $i$  at time  $t$ ;

$\alpha$  – previously defined;

$Z_{it}$  – stochastic variables considered  $N(0, \sigma_Z^2)$  and independent from  $X_i$ ;

$X_{it} = X_i e^{-\mu(t-T)}$ ;

$X_i$  – non-negative stochastic variables relating to production technical inefficiency and considered truncated to zero at distribution  $|N(0, \sigma_X^2)|$ ;

$\varphi$  – parameter to estimate.

Battese și Corra (1977) parameterisation is used; it replaces  $\sigma_Z^2$  and  $\sigma_X^2$  by

$$\sigma^2 = \sigma_Z^2 + \sigma_X^2 \text{ and } \varphi = \frac{\sigma_X^2}{\sigma_Z^2 + \sigma_X^2}.$$

Parameter  $\varphi$  belongs to the interval (0,1).

Also, the stochastic character of the production function can be tested. If the null hypothesis, when  $\phi$  equals zero, is accepted, it will indicate that  $\sigma_X^2$  is zero, and thus the term  $X_{it}$  can be taken out of the model, leaving a specification with parameters to be compatibly assessed by resorting to smallest normal differences.

**Model 2: Battese and Coelli (1995) specifications**

The empirical studies of Pitt and Lee (1981) have estimated the stochastic frontier and the efficiency at company level, by using, to this end, estimated functions.

Such issue has been also approached by Kumbhakar, Ghosh and McGukin (1991) and Reifschneider and Stevenson (1991) who propose stochastic frontier models where the inefficient effects ( $X_i$ ) are expressed as an explicit function of a vector of specific variables, at the company level. Battese and Coelli (1995) propose a model equivalent to the specification made by Kumbhakar, Ghosh și McGukin (1991), but the distributed efficiency is imposed.

The model specified by Battese and Coelli (1995) can be expressed as follows:

$$Q_{it} = \alpha u_{it} + (Z_{it} - X_{it}), \text{ cu } i = \overline{1, n} \text{ and } t = \overline{1, T} \tag{3}$$

where :

$Q_{it}$ ,  $u_{it}$  and  $\alpha$  are previously defined;

$Z_i$  - stochastic variables considered  $N(0, \sigma_Z^2)$  and independent from  $X_i$ ;

$X_i$  - non-negative stochastic variables relating to production technical inefficiency and frequently considered as  $N(m_{it}, \sigma_X^2)$  where:

$$m_{it} = z_{it} \delta \tag{4}$$

where:

$z_{it}$  - vector of type  $p \times 1$  that can influence the company, and  $\delta$  - vector of type  $1 \times p$  of the parameters to be estimated;

We will resort again to the parameterisation proposed by Battese and Corra (1977), by replacing  $\sigma_Z^2$  and  $\sigma_X^2$  by  $\sigma^2 = \sigma_Z^2 + \sigma_X^2$  și  $\phi = \frac{\sigma_X^2}{\sigma_Z^2 + \sigma_X^2}$ .

This specification of the model cumulates a number of specifications from other models, as well as special cases. If  $T=1$  and  $z_{it}$  takes value one and no other values, than the model may be reduced to the one given by Stevenson (1980).

**3. Cost function analysis**

All the above-mentioned specifications have been expressed in the terms of a production function, with  $X_i$  being construed as technical inefficiency effects,

determining the company to operate below the production stochastic frontier. If the specification of cost function stochastic frontier is wanted, the error term specification will be changed from  $(Z_i - X_i)$  to  $(Z_i + X_i)$ . For instance, this substitution will transform the functions defined in (1) into a cost function:

$$Q_i = \alpha u_i + (Z_i + X_i), \text{ with } i = \overline{1, N} \quad (5)$$

where:

$Q_i$  - production logarithm at the level of company  $i$ ;

$u_i$  - vector of type  $k \times 1$ ; it represents the input and outputs prices (transformations) of company  $i$ ;

$\alpha$  - vector of unknown parameters;

$Z_i$  - stochastic variables considered  $N(0, \sigma_Z^2)$  and independent from  $X_i$ ;

$U_i$  - non-negative stochastic variables relating to production technical inefficiency and considered  $|N(0, \sigma_X^2)|$ .

This cost function  $X_i$  defines herein how far downward the cost frontier the company operates. If the allotted efficiency is presumed,  $X_i$  is very close to the technical inefficiency cost. If such presumption is not undertaken, construing  $X_i$  as a cost function is less clear, with the two (technical and allotment) inefficiencies possibly involved.

The cost frontier (5) is identically proposed also by Schmidt and Lovell (1979). The log-probability function of the cost frontier is similar to the cost frontier one, save for several different signs. The log-probability function for the cost function is analogue to that of the models of Battese and Coelli (1992, 1995).

#### 4. Production frontier estimation by means of Cobb-Douglas, CES and translog functions

This approach starts from the assumption of a Cobb-Douglas production function:  $f(L, K) = AL^\alpha K^\beta$ .

This function  $f(\cdot, \cdot)$  is a power function with three parameters  $A$ ,  $\alpha$  and  $\beta$ ; therefore, it is log-linear (liner in the logarithm of the variables involved).

Here,  $A$  is a scaling factor, and  $\alpha$  and  $\beta$  are the elasticities corresponding to the two inputs considered. For the Cobb-Douglas function, the scale yield type is determined by the sum of the parameters representing elasticities, that is by  $\alpha + \beta$ , and the substitution elasticity is equal to 1.

Cobb-Douglas production function is used under an equivalent form, obtained by logarithmic transformation, that is:  $\ln Y = \ln A + \alpha \ln L + \beta \ln K$ . Parameters  $\alpha$  and  $\beta$  may be also construed as costs of the two production factors. If we denote by  $w$  the labour force unit price and by  $e$  the capital unit price, we could minimise the total

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production cost, depending on  $L$  and  $K$  inputs, for a production process described by the production function  $f$ .

Mathematically, this could be rendered as follows:

$$[\min]_{L,K} \{wL + eK\}$$

$$Y = f(L, K) = AL^\alpha K^\beta$$

The associated Lagrangian is:  $\Lambda = wL + eK - \lambda f(L, K)$ , and the necessary optimum conditions are:

$$\frac{\partial \Lambda}{\partial L} = w - \lambda \frac{\partial f}{\partial L} = 0 \quad \text{and} \quad \frac{\partial \Lambda}{\partial K} = e - \lambda \frac{\partial f}{\partial K} = 0$$

$$\text{By eliminating } \lambda, \text{ we obtain: } \frac{w}{\frac{\partial f}{\partial L}} = \frac{e}{\frac{\partial f}{\partial K}}$$

As we have, for Cobb-Douglas production function,  $\frac{\partial f}{\partial L} = \alpha \frac{f}{L}$  and

$$\frac{\partial f}{\partial K} = \beta \frac{f}{K}, \text{ the optimum necessary condition becomes } \frac{Lw}{\alpha} = \frac{Ke}{\beta}.$$

This relation expresses the fact that – in the production function – the cost of the two production factors (labour force cost  $Lw$  and capital cost  $Ke$ ) are proportional to Cobb-Douglas function parameters.

If we denote by  $p$  the production unit price, the whole production value resulted –  $Y$  is  $P = Yp$ . Therefore, another proportionality relation between the production value and the labour force (respectively capital) cost can be written down,

for instance:  $\frac{Lw}{\alpha} = \frac{P}{\alpha + \beta}$  or  $\frac{P}{L} = Cw$ , where  $C$  is a constant (respectively

$$\frac{P}{K} = C'e).$$

Under economic equilibrium conditions (minim cost), the ration  $P/L$  should be proportional to the labour force cost (cost of production factor  $L$ ). Yet, econometric researches performed, along years, for various industries, have inquired the previous statement. On the contrary, an appropriate adjustment of the ration  $P/L$  is given by the

relation:  $\frac{P}{L} = \log C + d \log w$ , where parameter  $d$  is significantly from zero.

Starting with experimental results, a production function compatible with them has been searched. A homogenous first degree production function has been looked for, resulting in:

**CES** (*Constant Elasticity of Substitution*), given by the expression:

$$f(L, K) = A[\delta L^{-\rho} + (1 - \delta)K^{-\rho}]^{-\frac{v}{\rho}}$$

Here,  $A$  is a scaling factor that could be deemed as „efficiency factor” as, for given  $L$  and  $K$ , the production obtained is proportional to it. Parameter  $v$  measures the scale yield, and  $\delta \in (0,1)$  is a parameter for revenue distribution between the two inputs. As for  $\rho$ , this is a substitution parameter, because  $\rho = \delta\sigma - 1$  where  $\sigma$  is the substitution elasticity.

In many respects, CES production function corresponds much better to the reality than Cobb-Douglas function. At the same time, the estimation of CES function parameters is more difficult, Cobb-Douglas function being, in this regard, preferred.

A more general production function than CES is **VES production function** (Variable Elasticity of Substitution), given by:

$$f(x_1, x_2) = Ax_1^{\gamma(1-\delta\rho)}[x_2 + (\rho-1)x_1]^{\delta\rho}$$

Here, the parameters are:  $A > 0$ ,  $\gamma > 0$ ,  $\delta\sigma \in (0,1)$  (the latter measuring the isoquant convexity). For this function, the substitution elasticity is  $\sigma = 1 + \frac{\alpha x_1}{x_2}$  and

obviously depends on the two inputs (from here comes also the function name).

**Translog production function** is used in practical applications due to its complex properties. It has the following form:

$$\ln Y = b_0 + b_1 \ln x_1 + b_2 \ln x_2 + \frac{1}{2}[b_{11} \ln^2 x_1 + b_{22} \ln^2 x_2 + b_{12} \ln x_1 \ln x_2]$$

and gives a second order local approximation, being fit for use in various situations. From this point of view, it has a flexible form.

Considering this latter issue, the residual factor has a very heterogeneous content; it might contain the effect of technological evolution, scale economy, inefficiency etc.

## 5. Cost frontier

The technical efficiency for company  $i$  during  $t$  is defined by:  $TE_{it} = e^{-u_i}$ , and the results of this value are programmed in Frontier.

The whole economic efficiency of company  $i$  is given by the following formula:  $EE_i = e^{-u_i}$ , where  $u_i$  is the effect of a non-negative inefficient cost.

This value is comprised between zero and one, and a similar modality can be predicted for describing the technical efficiency for the production stochastic frontier.

The whole economic efficiency of cost  $EE_i$  may be decomposed into its technical and allotment components, if the production function given by the estimated

cost function can be explicitly derived (this can be done when Cobb-Douglas formula is used, as it is dual in form).

For a simple example of such system, let's consider cost-translog function using one output and two inputs:

$$\ln c_i = \beta_0 + \beta_1 \ln w_{li} + \beta_2 \ln w_{2i} + \beta_3 \ln y_i + \beta_{12} \ln w_{li} \ln w_{2i} + \beta_{13} \ln w_{li} \ln y_i + \beta_{23} \ln w_{2i} \ln y_i + \frac{1}{2} [b_{11} \ln^2 w_{li} + b_{22} \ln^2 w_{2i} + b_{32} \ln^2 y_i] + v_i + u_i$$

## 6. Study of efficiency at industry level

### 6.1. Data source

The data set contains information taken from the accounting balance sheet and the profit and loss account of 20 companies operating in the construction field, for the period 2006-2010. This information has been taken by means of the site [www.rasd.ro](http://www.rasd.ro), from where the currently tradable companies have been selected.

The European currency depreciation and the increase of the price of utilities strongly affect the Romanian construction industry.

Out of more than 7.600 companies operating in the construction industry, just 270 are large companies and only these ones have chances to extend their lifetime on the market.

The business in the construction industry will be of about 8,5 billion Euro in 2011, two billion Euro less as compared to 2010.

### 6.2. Description of variables

The Input variables used in this analysis are:

- fixed assets, expressed in RON;
- inventories, RON;
- number of employees, expressed in persons, representing the number of employees of these companies, per year.

The Output variables used in this analysis are:

- operating revenues;
- net profit.

### 6.3. Description of data

All data are expressed in real time, for this purpose being used, as deflator, the Consumption Price Index relating to 1991.

## 6.4. DEA results

The elements of Data Envelopment Analysis methods are estimated by using DEAP 2.1 software, programming tool conceived by Tim Coelli (1996a). The company efficiency scores are computed by using the two hypotheses: scale constant return – CRS and scale variable return –VRS.

In order to analyse the above-mentioned data, by means of DEAP software, a data file and an instruction file have been constructed. All files containing data, instructions and results are text-type files.

## 6.5. Complex analysis for the case with two outputs and three inputs

### 6.5.1. VRS Input Orientation

The data file for this case was named OOIII.DTA. This file contains five observations of the two outputs and three inputs. The Output quantities are listed in the first two columns and the Inputs in the next three columns.

The file with instructions, OOIII.INS, contains the names of the instructions and data files. In the next four lines, the following are rendered: number of companies (20); number of time periods (5); number of outputs (2) – due to the inclusion of a new output in the analysis; and number of inputs (3). The following three lines contain the specification «1» for VRS method; «0» for input orientation and «0» for DEA standard model estimation.

After having created the two files, DEAP programme has been run. The name of the file with instructions OOIII.INS was introduced. The programme centralised the results in a file named OOIII.OUT.

#### *Interpretation of results*

DEA results – VRS Input Oriented are presented in the following table:

*Table 1*

**DEA results – VRS Input Orientation**

Company	CRS TE	VRS TE	Scale E	
1	0.440	0.459	0.958	drs
2	0.497	0.499	0.995	drs
3	0.262	1.000	0.262	irs
4	1.000	1.000	1.000	-
5	0.590	0.612	0.964	irs
6	1.000	1.000	1.000	-
7	1.000	1.000	1.000	-
8	0.503	0.763	0.659	drs

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Company	CRS TE	VRS TE	Scale E	
9	0.294	0.314	0.937	irs
10	<b>1.000</b>	<b>1.000</b>	<b>1.000</b>	-
11	0.955	0.979	0.975	irs
12	0.751	0.756	0.993	irs
13	0.060	0.115	0.518	irs
14	<b>1.000</b>	<b>1.000</b>	1.000	-
15	<b>1.000</b>	<b>1.000</b>	1.000	-
16	0.951	<b>1.000</b>	0.951	drs
17	0.253	0.312	0.812	irs
18	0.625	0.657	0.952	drs
19	0.314	0.361	0.872	irs
20	<b>1.000</b>	<b>1.000</b>	1.000	-
<b>Mean</b>	0.675	0.741	0.892	

It can be seen that companies 4, 6, 7, 10, 14, 15 and 20 are the only efficient companies, when CRS method is applied, and companies 3, 4, 6, 7, 10, 14, 15, 16 and 20, when VRS method is applied. 5 companies register scale decreasing return, 8 companies register scale increasing return and 7 companies are efficient.

Different efficiency value computation can be illustrated by resorting to companies 1, 2, 5, 8, 9, 11, 12, 13, 17, 18, 19, companies inefficient in both methods: CRS and VRS. For instance, for company 8, CRS technical efficiency is 0.503; VRS technical efficiency is 0.763 and scale efficiency is 0.659, computed as ratio between the two terms. The technical efficiency shows us that the company may reduce the input level by 23.70% and may obtain the same level of output. As it can be seen, company 8 registers scale decreasing return.

If we do compare the two analyses, we observe just small variations in results, they being, in essence, the same. Thus, the influence of the second output is not significant.

The information regarding the values of inputs and outputs – slacks represent the points of projection on the efficiency frontier, and indicate how much the output should increase so that the input value might remain the same. The only difference from the analysis corresponding to one output is the occurrence of the second one.

### 6.5.2. VRS Output Orientation

The data file for this case was named OOIIIo.DTA. This file contains five observations of the two outputs and three inputs. The Output quantities are listed in the first two columns and the Inputs in the next three columns.

In the file with instructions, OOIIIo.INS, the only modification is given by value «1» indicating an output orientation.

After having created the two files, DEAP programme has been run. The name of the file with instructions OOIIIo.INS was introduced. The programme centralised

the results in a file named OOOIo.OUT. It is to be noted that, when VRS option is selected, DEAP programme computes the technical efficiency corresponding to the CRS and VRS methods and the scale efficiency.

*Interpretation of results*

DEA results – VRS Output Oriented are presented in the following table:

*Table 2*

**DEA results – VRS Output Orientation**

Company	CRS TE	VRS TE	Scale E	
1	0.440	0.512	0.859	drs
2	0.497	0.549	0.905	drs
3	0.262	1.000	0.262	irs
4	1.000	1.000	1.000	-
5	0.590	0.592	0.996	drs
6	1.000	1.000	1.000	-
7	1.000	1.000	1.000	-
8	0.503	0.937	0.536	drs
9	0.294	0.366	0.803	drs
10	1.000	1.000	1.000	-
11	0.955	0.977	0.977	irs
12	0.751	0.759	0.989	drs
13	0.060	0.085	0.701	drs
14	1.000	1.000	1.000	-
15	1.000	1.000	1.000	-
16	0.951	1.000	0.951	drs
17	0.253	0.254	0.997	drs
18	0.625	0.665	0.940	drs
19	0.314	0.319	0.987	irs
20	1.000	1.000	1.000	-
Mean	0.675	0.751	0.895	

Table 3 centralises the data obtain in the two cases: input orientation, using the inputs and outputs of the 20 companies all over five periods (2006-2010). The first column indicates the results obtained after having applied the scale constant return method (CRS), the second column presents the results obtained after having applied the scale variable return method and the last column centralises the scale efficiency data. The efficiency mean, using CRS, is 0.975, the scale efficiency mean is 0.89 and the efficiency mean, using VRS differs a little bit between the two orientations, amounting to 0.741, respectively 0.751.

Table 3

**DEA Multistage – Input Output Orientation**

	Input Orientation			Output Orientation		
	CRS	VRS	SE	CRS	VRS	SE
Mean	0.675	0.741	0.892	0.675	0.751	0.895

Table 2 reveals that companies 4, 6, 7, 10, 14, 15 and 20 are the only efficient companies, when CRS method is applied, and companies 3, 4, 6, 7, 10, 14, 15, 16 and 20 when VRS method is applied. 10 companies register scale decreasing return, 3 companies register scale increasing return and 7 companies are efficient.

Different efficiency value computation can be illustrated by resorting to companies 1, 2, 5, 8, 9, 11, 12, 13, 17, 18, 19, companies inefficient in both methods: CRS and VRS. For instance, for company 8, CRS technical efficiency is 0.503; VRS technical efficiency is 0.937 and scale efficiency is 0.536, computed as ratio between the two terms. The technical efficiency shows us that the company may reduce the output level by 6.3% and may produce the same level of input. As it can be seen, company 8 registers scale decreasing return.

The information regarding the values of inputs and outputs – slacks represent the coordinates of the points of projection on the efficiency frontier, and indicate how much the output should increase so that the input value might remain the same.

**6.5.3. CRS VRS Input Orientation by years**

Hereinafter, we will analyse the efficiency indicators, by using CRS and VRS methods, for each and every year, for the period 2006-2010. To this end, DEA multistage option with input orientation was selected. A DTA type file was created, containing data corresponding to the 20 companies, for each of the five analysed years. The results are centralised and rendered in table 4.

As it can be seen, with CRS assumption, the efficiency level remained constant during the first two years, then it increased in 2008 from 0.675 to 0.808. Since 2008, there was a decrease, reaching in 2009 a value of 0.795 and in 2010 a value of 0.705. The scale efficiency increases from 0.89 in 2006 to 0.963 in 2009, then it decreases to 0.891 in 2010. The property of scale decrease return changes in time, so that in 2006 5 companies show this property, in 2007 3 companies, in 2008 2 companies, in 2009 5 companies, and, in 2010 7 companies. The number of companies with scale increase return also changes. Thus, during the first year 8 companies show this property, the next year 10 companies, in 2008 7 companies, in 2009 5 companies and in 2010 6 companies. In 2006, 2007 7 companies are efficient, in 2008 11 companies, in 2009 10 companies, decreasing in 2010 to 7 efficient companies. It can be also seen that companies 4, 14, 15, 20 are efficient all over the five years, company 16 manifested a scale decrease return during the first two years, becoming efficient the next three years.

Analysing the slacks contained in table 5, assuming the two methods, the highest values are reflected for the first two inputs: fixed assets and inventories. Therefore, the companies could reach efficiency by decreasing the level of inputs by the values rendered in the table.

Table 4

**DEA Multistage by years – Input Orientation**

Model	Years	Mean
CRS	2006	0.675
	2007	0.675
	2008	0.808
	2009	0.795
	2010	0.705
VRS	2006	0.741
	2007	0.723
	2008	0.856
	2009	0.818
	2010	0.772
SE	2006	0.892
	2007	0.916
	2008	0.933
	2009	0.963
	2010	0.891

Table 5

**Slacks – Input Orientation**

Input variables				
		Fixed assets	Inventories	No. of employees
CRS	2006	34874.795	715046.004	13.146
	2007	16234.769	1269495.253	38.954
	2008	357957.110	686931.106	0.141
	2009	254894.253	833450.265	0.000
	2010	139004.419	1004435.394	0.000
VRS	2006	3427429.133	238938.535	22.510
	2007	702715.552	408365.393	4.935
	2008	950026.331	874300.042	0.592
	2009	1608000.899	497774.655	0.526
	2010	1990681.737	911901.254	0.000

## 7. Conclusions

This study renders the efficiency analysis performed by using Data Envelopment Analysis (DEA) - input orientation method, analysis made for each of the five years considered, from 2006 to 2010. The technical efficiency estimation has been also applied in the following cases: input-output orientation, by resorting to VRS method, all over the period 2006-2010, with two outputs and three inputs.

The results reveal that the efficiency level is quite low for certain companies. Therefore, there should be a higher interest for increasing the efficiency level in the construction industry. Besides, there is a difference between the technical efficiency values for the periods 2006-2008 and 2009-2010. If during the first period, there is an increase of the efficiency level, the following period, a decreasing trend is registered.

At company level, the number of the efficient ones is quite low, out of 20, only 7 being efficient. This result reflects the current situation - out of 7.600 companies activating in the industry field, just 270 are large companies and only these ones have chances to extend their lifetime on the market. The resulting efficient companies are: 4, 6, 7, 10, 14, 15 and 20.

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